

### 3/7 Cuts

Wednesday, March 7, 2018 6:44 PM

A  $s-t$  cut of a graph  $G$  is as follows

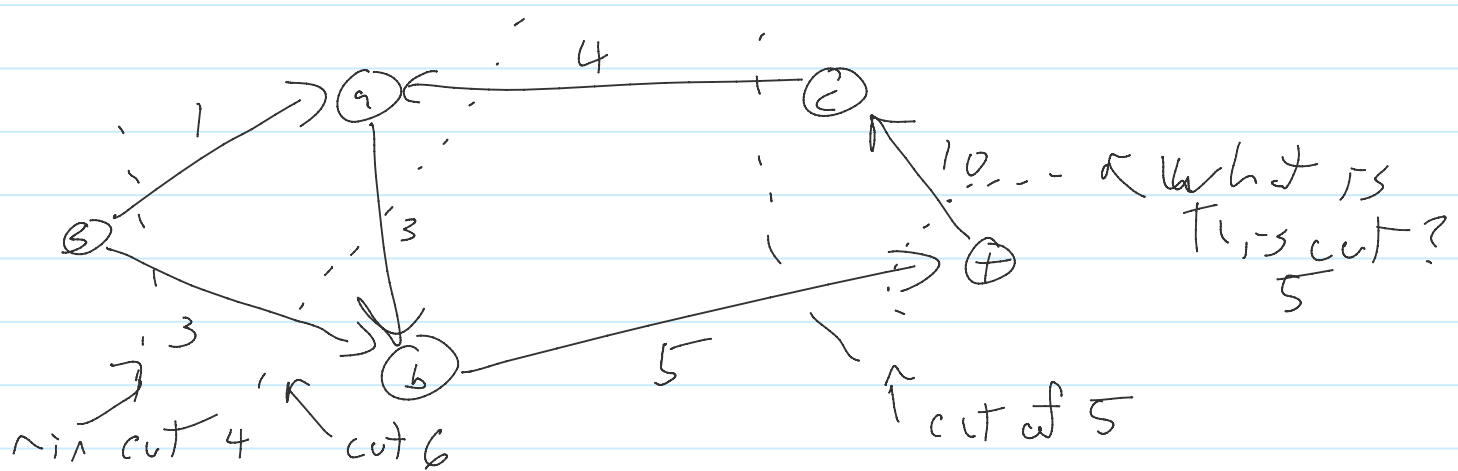
Let  $S, T$  be partitions of  $V(G)$

Let  $s \in S$  &  $t \in T$  the cut

is set of edges traversing  $S \rightarrow T$

$$\{ (u, v) \in E : u \in S \wedge v \in T \}$$

$\uparrow$  from  $S$  to  $T$



$$S' = \{s, a\}$$

$$T' = \{b, c, t\}$$

$$\text{Cut} = \{ (s, b), (a, b) \}$$

$$\sum_{e \in \text{cut}} e.w = \text{cut size, effort, or w/e you call it}$$

Minimum cut of a graph is equal to the maximum flow of a graph.

Let  $A$  contains  $s$  but  $t \notin A$

the flow out of  $A$  is equal flow into  $A^c$

Run FFA on  $G$  and get the  $G_r$

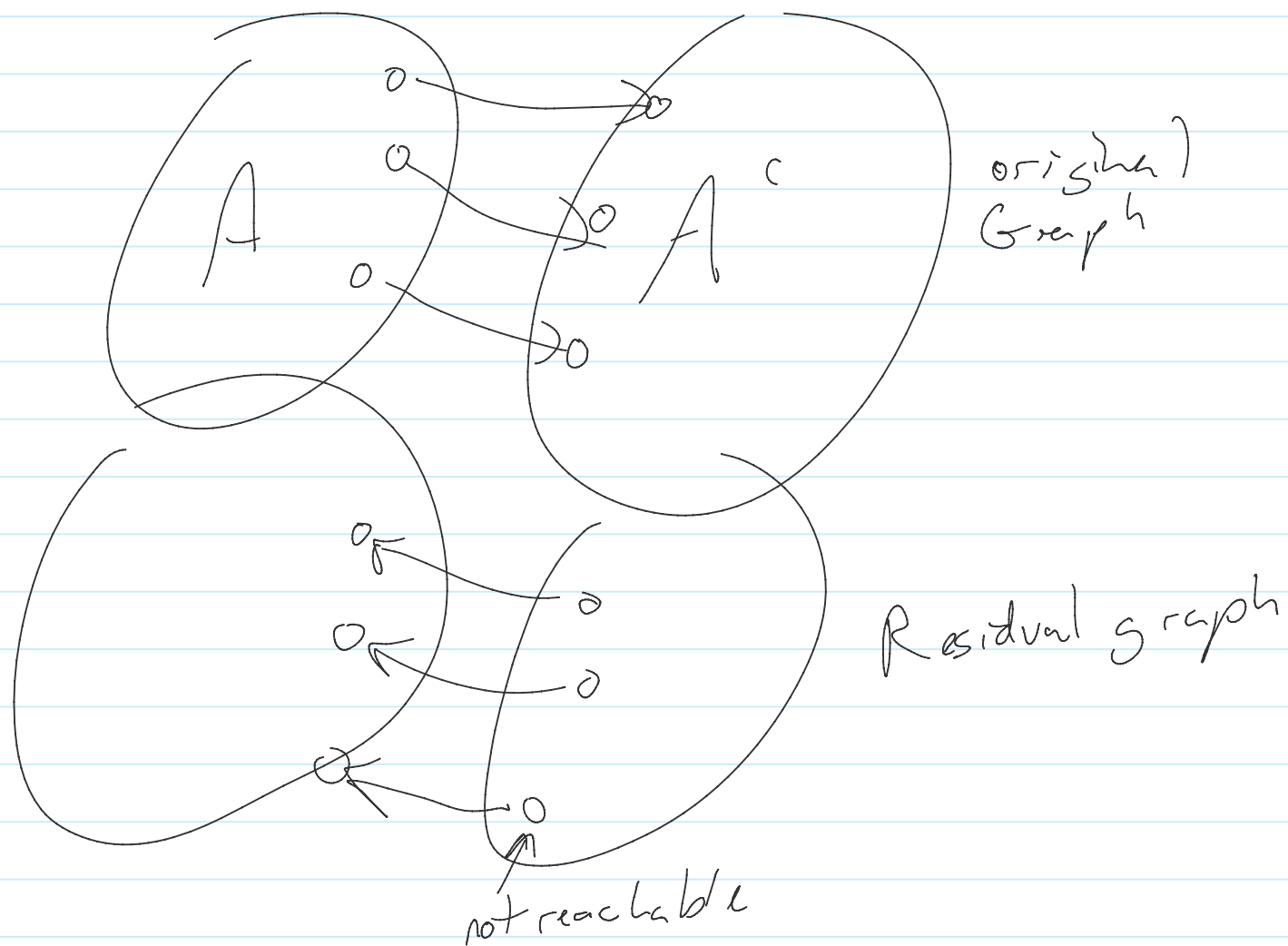
Let  $A$  be the set of nodes reachable by

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$s$  in  $G$ . Let  $A^c = V \setminus A$

All edges leaving  $A$  must be saturated  
(flow = capacity)  
in  $G$

if not then

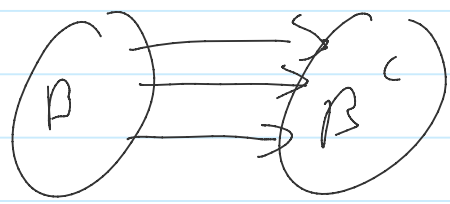


Each edge from  $A^c$  to  $A$  in original graph must have 0 flow otherwise in residual graph a edge with reverse direction would exist and nodes from  $A^c$  would be reachable by  $s$ .

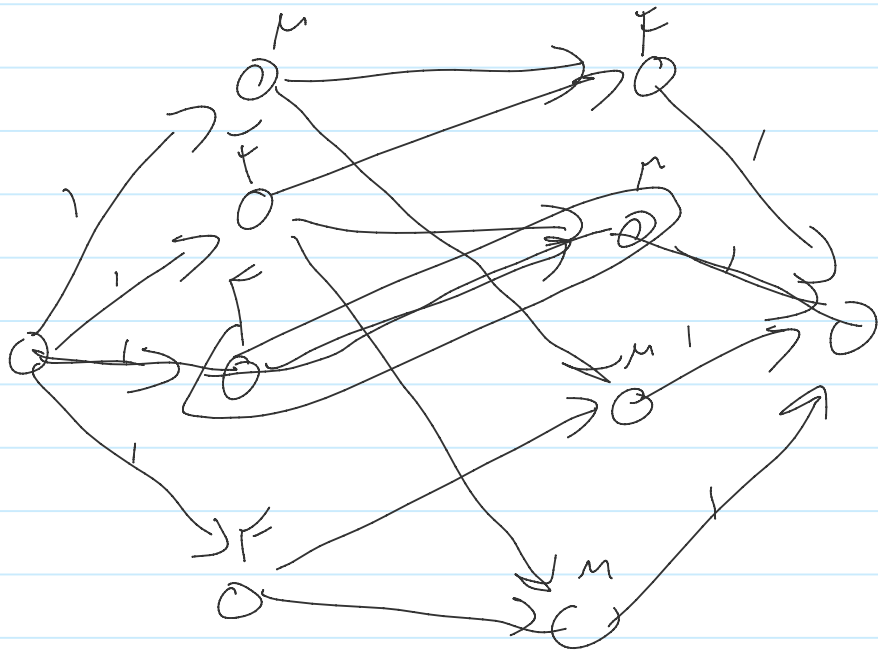
we have a cut from  $s$  to  $t$  using the forward edges in the original graph

$A$  and  $A^c$  is a cut

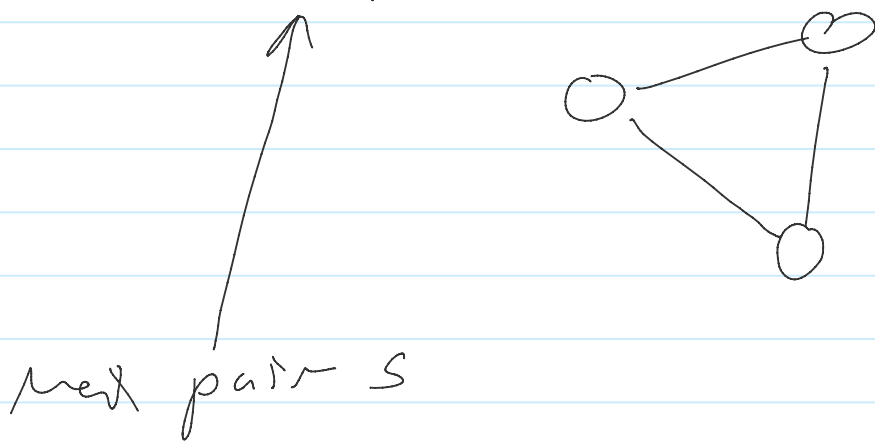
Suppose we have a smaller cut than the max flow.



flow out of  $B$  is equal to flow into  $B^c$   
 new min cut creates a smaller max flow  
 this creates a contradiction.

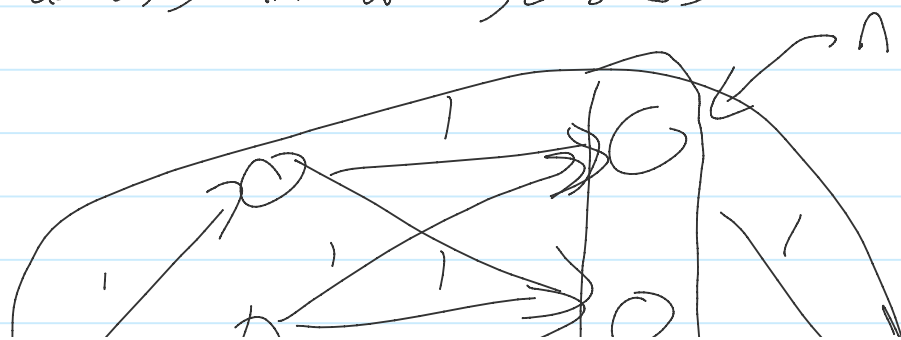


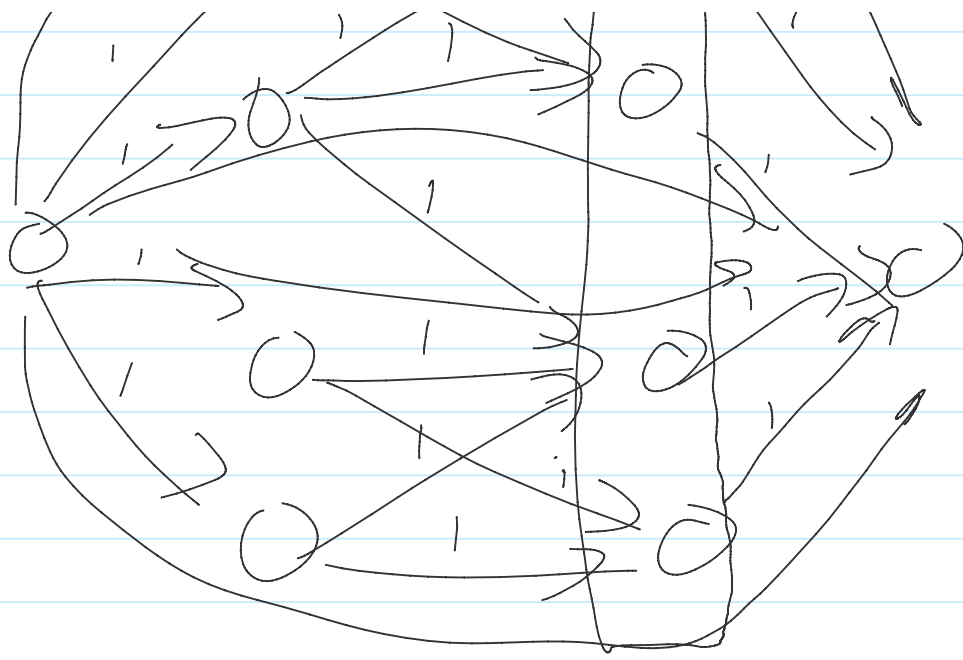
only 1 partner



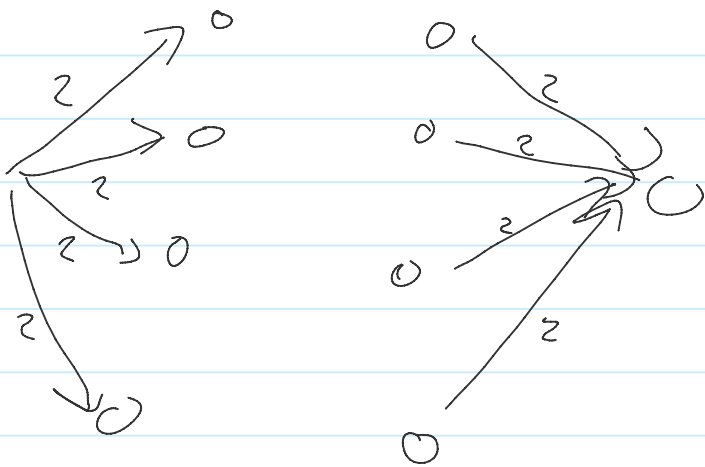
Speed dating

goal find maximum number of arrangements  
 where each person has 1 partner and each  
 does not have the same partner twice  
 across all arrangements



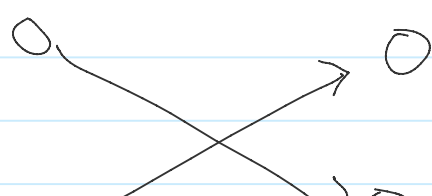
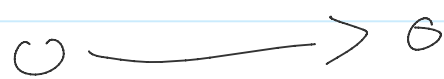
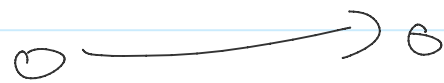
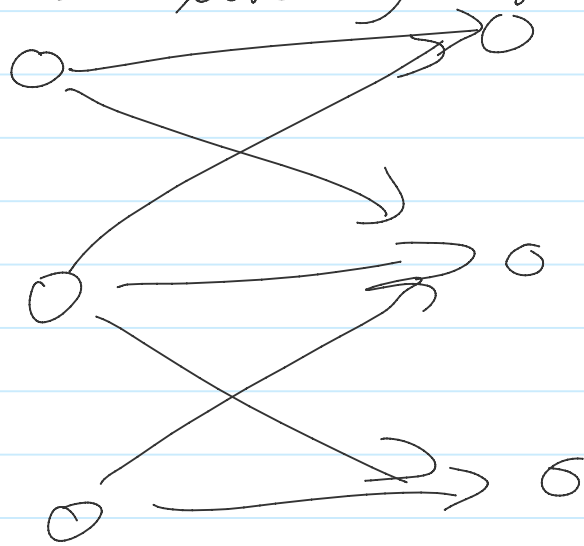


$n$



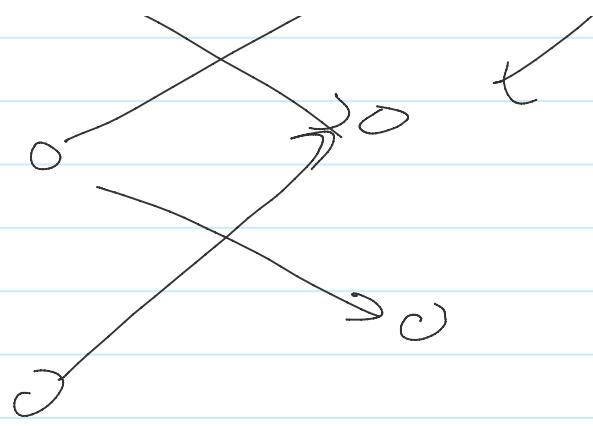
Greedy concept for extracting matching.

find a maximum matching and remove it (repeat).



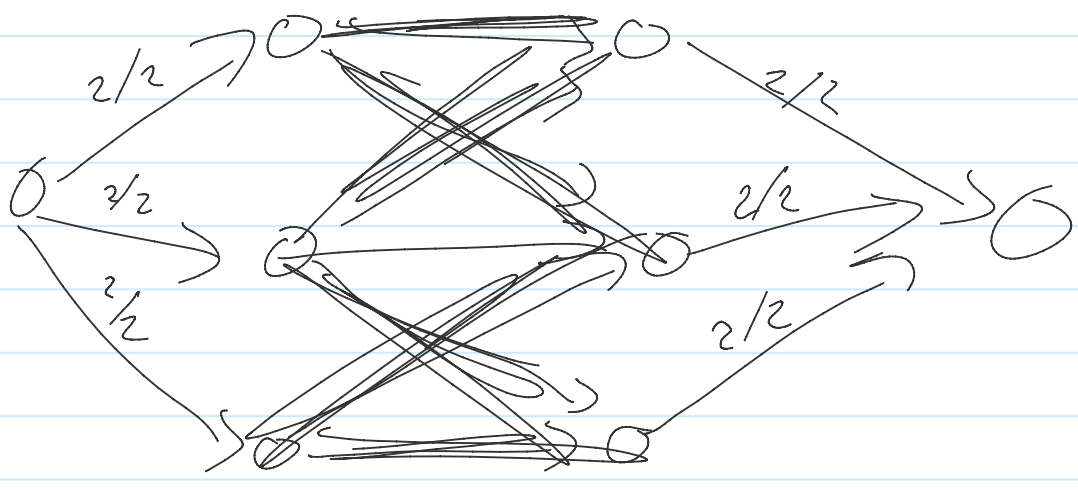
matching

Residual graph  
beal



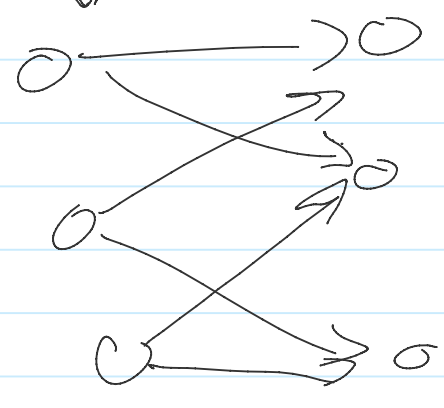
bad  
This greedy does not work!

Better greedy method



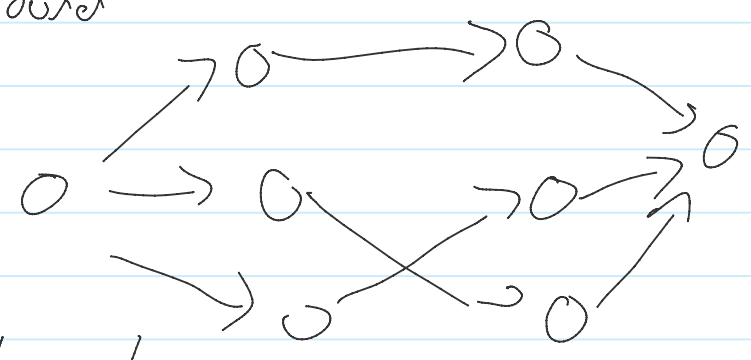
remove unused edges

New graph

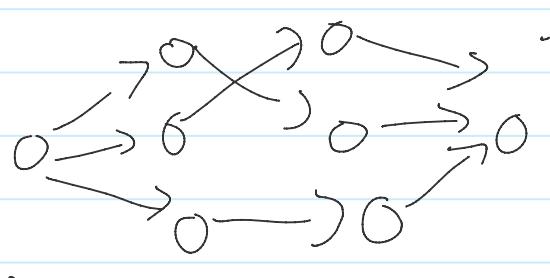


Then perform single maximum matching repeatedly to find the different rounds.

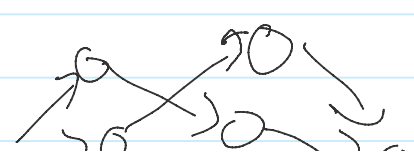
First round

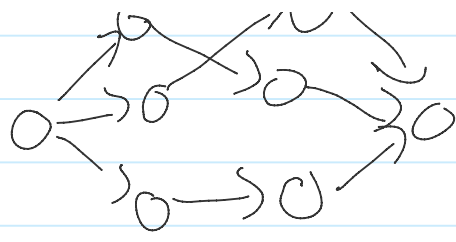


residual graph



second round





residual graph

